A

1016

Sr. No. ....

Paper Code : 16

MATHEMATICAL SCIENCES [Paper-III]

Signature and Name of Invigilator

1. (Signature) ______________________
   (Name) ______________________
2. (Signature) ______________________
   (Name) ______________________

OMR Sheet No. : ....

(To be filled by the candidate)

Roll No. | | | | | | | | | |

(In Figures as per admission card)

Roll No. ______________________

(In words)

Time : 2½ Hours] **[Maximum Marks : 150**

Number of Pages in this Booklet : 24 **Number of Questions in this Booklet : 75**

Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of seventy five multiple-choice type of questions.
3. At the commencement of examination, the question bookelt will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Fault booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (ii) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the oval as indicated below on the correct response against each item.
   Example : (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the items are to be indicated in the Answer Sheet given inside the Paper I Booklet only. If you mark at any place other than in the ovals in the Answer Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the test booklet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test question booklet and OMR Answer sheet to the invigilators at the end of the examination compulsorily and must not carry it with you outside the Examination Hall.
10. Students can take home carbon copy of this OMR answer sheet.
11. Use only Blue/Black Ball point pen.
12. Use of any calculator or log table etc., is prohibited.
13. There is no negative marks for incorrect answers.

परीक्षार्थियों के लिए निर्देश

1. पहले पृष्ठ के ऊपर नियत स्थान पर अपना रोल नम्बर लिखिए।
2. इस प्रश्न-पत्र में पिच्हेत्तर बहुविकल्पीय प्रश्न हैं।
3. परीक्षा प्रारम्भ होने पर, प्रश्न-पुस्तिका आपको दे दी जायेगी। पहले पाँच मिनट आपको प्रश्न-पुस्तिका खोलने तथा उसकी निम्नलिखित जाँच के लिए दिये जायेंगे, जिसकी जाँच आपको अवश्य करनी है :
   (i) कवर पृष्ठ पर छपे निर्देशानुसार प्रश्न-पुस्तिका के पृष्ठ तथा प्रश्नों की संख्या को अच्छी तरह चैक कर लें कि ये पूरे हैं। दोषपूर्ण पुस्तिका जिनमें पृष्ठ/प्रश्न कम हों या दुबारा आ गये हों या सीरियल में न हों अर्थात् किसी भी प्रकार की त्रुटिपूर्ण पुस्तिका स्वीकार न करें तथा उसी समय उसे लौटाकर उसके स्थान पर दूसरी सही प्रश्न-पुस्तिका ले लें। इसके लिए आपको पाँच मिनट दिये जायेंगे। उसके बाद न तो आपको प्रश्न-पुस्तिका वापस ली जायेगी और न ही आपको अतिरिक्त समय दिया जायेगा।
   (ii) इस जाँच के बाद OMR पत्रक की क्रम संख्या इस प्रश्न-पुस्तिका पर अंकित कर दें।
4. प्रत्येक प्रश्न के लिए चार उत्तर पत्रक विकल्प (A), (B), (C) तथा (D) दिये गये हैं। आपको सही उत्तर के दीर्घवृत्त को पेन से भरकर काला करना है जैसा कि नीचे दिखाया गया है।
   उदाहरण : (A) (B) ● (D)
   जबकि (C) सही उत्तर है।
5. प्रश्नों के उत्तर केवल प्रश्न पत्र I के अन्दर दिये गये उत्तर-पत्रक पर ही अंकित करने हैं। यदि आप उत्तर पत्रक पर दिये गये दीर्घवृत्त के अलावा किसी अन्य स्थान पर उत्तर चिह्नांकित करते हैं, तो उसका मूल्यांकन नहीं होगा।
6. अन्दर दिये गये निर्देशों को ध्यानपूर्वक पढ़ें।
7. कच्चा काम (Rough Work) इस पुस्तिका के अन्तिम पृष्ठ पर करें।
8. यदि आप उत्तर-पुस्तिका पर अपना नाम या ऐसा कोई भी निशान करते हैं तो परीक्षा के लिये अयोग्य घोषित कर दिये जायेंगे।
9. आपको परीक्षा समाप्त होने पर प्रश्न-पुस्तिका एवं OMR उत्तर-पत्रक निरीक्षक महोदय को लौटाना आवश्यक है और परीक्षा समाप्ति के बाद उसे अपने साथ परीक्षा भवन से बाहर न लेकर जायें।
10. परीक्षा समाप्ति पर परीक्षार्थी OMR उत्तर-पत्रक की कार्बन कापी अपने साथ ले जा सकते हैं।
11. केवल नीले/काले बाल प्वाईंट पेन का ही इस्तेमाल करें।
12. किसी भी प्रकार का संगणक (केलकुलेटर) या लाग टेबल आदि का प्रयोग वर्जित है।
13. गलत उत्तरों के लिए कोई अंक काटे नहीं जाएँगे।

Paper Code : [16]

Paper-III [MATHEMATICAL SCIENCES]

Note : • **This paper contains Seventy Five (75) multiple choice questions, each question carrying two (2) marks.**

नोट : • **इस प्रश्नपत्र में पिच्हेत्तर (75) बहुविकल्पीय प्रश्न हैं। प्रत्येक प्रश्न के दो (2) अंक हैं।**

1. Let A be any set and 2^A denote the power set of A. Which of the following statements is not true ?

(A) If A is countable, then 2^A is countable.

(B) If 2^A is countable, then A is countable.

(C) If A is uncountable, then 2^A is uncountable.

(D) If 2A is uncountable, then A is either countable or uncountable.

2. Which of the following statements is true ?

(A) Every infinite subset of real numbers has a limit point.

(B) Every bounded infinite subset of real numbers has a unique limit point.

(C) A bounded infinite subset of real numbers must have atleast one limit point.

(D) Any infinite subset of real numbers has necessarily infinitely many limit points.

3. Let $f(x) = e^{-x^2}$ and $g(x) = \frac{1}{Hx^2}$

Which of the following is true ?

(A) $f(x) \geq g(x)$ for all $x \geq 0$ (B) $f(x) \leq g(x)$ for all x

(C) $f(x) - g(x)$ changes sign finitely many times as x varies over $[0, \infty]$.

(D) $f(x) - g(x)$ changes signin finitely many times as x varies over $[0, \infty]$.

4. Let $a_n = (-1)^n\left(\frac{n+1}{2n}\right)$ for each $n \geq 1$. Then :

(A) $\limsup\{a_n\} = \liminf\{a_n\} = \frac{1}{2}$ (B) $\limsup\{a_n\} = \frac{1}{2}$ and $\liminf\{a_n\} = -\frac{1}{2}$

(C) $\limsup\{a_n\} = \frac{1}{2}$ and $\liminf\{a_n\} = -\frac{2}{3}$ (D) $\limsup\{a_n\} = \liminf\{a_n\} = 0$

5. Let $\{a_n\}$ be a monotonically decreasing sequence of positive numbers such that $\sum_{n=1}^{\infty} a_n < \infty$.

Then :

(A) $\sum_{n=1}^{\infty} a_n < \infty$

(B) $n a_n \to \infty$ as $n \to \infty$

(C) $n a_n \to 0$ as $n \to \infty$

(D) $\sum_{n=1}^{\infty} n a_n$ dimvrges

6. Let f : $\mathbb{R} \to \mathbb{R}$ be a function such that f(x + y) = f(x).f(y) ∀ x, y ∈ $\mathbb{R}$. If further f is continuous at 0, then :

(A) $\lim_{x \to 0} f(x) = 0$

(B) $\lim_{x \to 0} f(x) = 1$

(C) $\lim_{x \to 0} f(x) \neq 0$

(D) $\lim_{x \to 0} f(x) = 0$ or $\lim_{x \to 0} f(x) = 1$

7. Let f : [0, ∞] R be given by $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

Then :

(A) *f* is Riemann integrable

(B) *f* is Lebesgue integrable

(C) The improper Riemann integral of *f* exists

(D) The improper Riemann integral of *f* does not exist

8. Let $f_n(x) = \frac{1}{nx+1}$, for all $n \geq 1$. Then the sequence $\{f_n\}$ converges :

(A) uniformly to zero on [0, 1].

(B) uniformly to zero on (0, 1].

(C) pointwise to zero on (0, 1].

(D) pointwise to zero on [0, 1].

9. Let $g(x) = \int_0^{x^2} t \log t \, dt$. Then g(x) is differentiable and g′(x) is equal to :

(A) x log x

(B) $x^2 \log x^2$

(C) $x^3 \log x^3$

(D) $2x^3 \log x^2$

10. Let us denote by P(R), the power set of ℝ, and by M, the family of all lebesgue measurable set in R. If $\overline{\overline{X}}$ stands for the cardinal number of the set X, then :

(A) M = P(ℝ)

(B) $M \subset P(R)$ and $\overline{\overline{M}} < \overline{\overline{P(R)}}$

(C) $M \subset P(R)$ and $\overline{\overline{M}} = \overline{\overline{P(R)}}$

(D) $M \subset P(R)$ and $\overline{\overline{M}} \leq \overline{\overline{P(R)}}$

11. Let $f(x,y) = \begin{cases} \frac{xy}{(x^2+y^2)^{1/2}}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{otherwise} \end{cases}$ Then :

(A) f is continuous at (0, 0)

(B) f is not continuous at (0, 0) but $\lim_{(x,y)\to(0,0)} f(x,y)$ exists.

(C) f is differentiable at (0, 0).

(D) $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

12. Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be defined as $f(x, y, z) = (2x + 3y, 2y - 3z)$.

Then

(A) f is differentiable at each point and f' is constant.

(B) f is differentiable at each point but f' is not constant

(C) f is differentiable but f' is not a linear transformation.

(D) f is differentiable at (0, 0, 0) only but $f'(0, 0, 0) \neq f$.

13. Let (X, d) be a metric space for $x, y \in X$ define $d_1(x,y) = \frac{2d(x,y)}{1+2d(x,y)}$

Then :

(A) (X, d_1) is not a metric space

(B) (X, d_1) is a metric space in which every subset of X is bounded.

(C) (X, d_1) is a metric space in which every subset of X is closed.

(D) (X, d_1) is a metric space in which every subset of X is bounded and closed.

14. Let $(X, \| \|)$ be a normed linear space. For $x, y \in X$ define $d(x, y) = \|x-y\|$. Then :

(A) d is a translation invariant metric on X.

(B) d is a metric on X but is not translation in-variant.

(C) $d(\alpha.x, \alpha.y) = \alpha\, d(x, y)\ \forall\, x, y \in X$ and for each real number α.

(D) d is not a metric on X.

15. Let $(X, \| \|)$ be a normed linear space and $B = \{x \in X : \|x\| \leq 1\}$. Then :

(A) B is bounded and closed but never compact.

(B) B is bounded but not closed.

(C) B is bounded, closed and compact.

(D) B is compact if and only if X is finite dimensional.

16. If $\{V_1, V_2, ... V_n\}$ is linearly independent set of a vector space V and $T : V \to W$ is an injective (one-one) linear transformation then :

(A) $\{T(V_1), ...T(V_n)\}$ is linearly dependent in W

(B) $\{T(V_1) - T(V_2), T(V_2) - T(V_3), ... , T(V_{n-1}) - T(V_n)T(V_n)\}$ is linearly independent in W

(C) $\{T(V_1), ..., T(V_n)\}$ is neither linearly dependent nor linearly independent.

(D) $\{T(V_1), ..., T(V_n)\}$ is not a basis of W.

17. If u and v are elements of inner product space such that $\|u\| = 3$, $\|u + v\| = 4$, $\|u - v\| = 6$ then:

(A) $\|V\| = 17$ (B) $\|V\| = 19$

(C) $\|V\| = \sqrt{17}$ (D) $\|V\| = \sqrt{19}$

18. Define $T : R^2 \to R^2$ by $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2)$. The matrix of T^{-1} relative to the standard basis R^2 is :

(A) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ (D) $\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

19. The number of vectors in an n dimensional vector space over a finite field with q elements is :

(A) n^q (B) q^n

(C) nq (D) none of these

20. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (x, y, 0)$ and $S : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $S(x, y) = (2x, 3y)$, be linear transformations on real vector spaces $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively. Then which of the following is correct ?

(A) T and S both are singular. (B) T and S both are non singular.

(C) T is singular and S is non-singular (D) S is singular and T is non-singular

21. Let C be the real vector space of complex numbers and T : C → C be a linear transformation given by $T(z) = \overline{Z}\ \forall Z \in C$. Then

(A) T is one-one but not onto.

(B) T is onto butnot one-one.

(C) T is one-one and onto.

(D) T is neither one-one nor onto.

22. If S and T are linear operators on a vector space V then :

(A) ST – TS can never be I (identity map)

(B) ST - TS can be equal to I

(C) trace (ST – TS) ≠ 0

(D) trace (ST – TS) is always positive

23. Let A be the 4 × 4 real matrix :

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -2 & -2 & 2 & 1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$$

Then :

(A) Characteristic polynomial for A is $x^2\ (x-1)^2$

(B) Cahracteristic polynomial for A is $x^3(x-1)$

(C) $x^2(x-1)^2$ is a minimal polynomial for A.

(D) (A) and (C)

24. Which of the following is the set of orthogonal vectors in R^3 ?

(A) {(3, 0, 4), (–1, 0, 7), (2, 9, 11)}

(B) {(1, 0, 1), (1, 0, –1), (0, 3, 4)}

(C) {(3,0,4), (–4, 0, 3), (0, 1,0)}

(D) $\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),(1,0,0),(0,0,1)\right\}$

25. Let α be a primitive fifth root of unity. Define :

$$A = \begin{bmatrix} \alpha^{-2} & 0 & 0 & 0 & 0 \\ 0 & \alpha^{-1} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & \alpha^2 \end{bmatrix}$$

For a vecor $\bar{V} = (V_1, V_2, \ldots\ldots, V_5) \in R^5$, define $\|\bar{V}\|_A = \sqrt{|\bar{V} A \bar{V}^T|}$, where $\bar{V}^T$ is transpose of $\bar{V}$. If

$\bar{W} = (1, -1, 1, 1, -1)$, then $\|\bar{W}\|_A$ equals :

(A) 0 (B) 1

(C) –1 (D) 2

26. Let a_1 and a_2 be complex numbers. The radius of convergence of the power series :

$$\sum_{n=0}^{\infty} \left(a_1^n + a_2^n\right) z^n$$

is :

(A) $\frac{1}{|a_1|} + \frac{1}{|a_2|}$ (B) $\max\left(\frac{1}{|a_1|}, \frac{1}{|a_2|}\right)$

(C) $\min\left(\frac{1}{|a_1|}, \frac{1}{|a_2|}\right)$ (D) None of the above

27. The residue of the function :

$$f(z) = \frac{2z}{(z+4)(z-1)^2}$$

at the point z = 1 is :

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

(C) $\frac{8}{25}$ (D) $\frac{4}{25}$

28. The bilinear transformation given by

$$w = e^{i\theta}\left(\frac{z-p}{pz-1}\right), |p| < 1$$

maps $|z| < 1$ onto :

(A) $|w| < 1$ (B) $|w| < 1$

(C) $|w| > 1$ (D) $|w| \geq 1$

29. For the function f given by

$$f(z) = \frac{1}{z}\cosh\frac{1}{z}$$

the point at z = 0 is :

(A) a removable singularity (B) a pole

(C) an essential singularity (D) none of the above

30. For the function f given by

$f(z) = z^2 e^{-z^4}$

the point at z = 0 is :

(A) a removable singularity (B) a pole

(C) an essential singularity (D) none of the above

31. The series $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$ converges for :

(A) $|z| < 1$

(B) $|z| \leq 1$

(C) $|z| \leq 1$ except at $z = 1$

(D) all $z \in C$

32. Let f be analytic in $|z| \leq R$ and $|f(z)| \leq M$. Then :

(A) $|f(z)| \leq \frac{M}{R}|z|$

(B) $|f(z)| < \frac{M}{R}|z|$

(C) $|f(z)| = \frac{M}{R}|z|$

(D) None of the above

33. A group is simple, then its order may be :

(A) 20

(B) 30

(C) 60

(D) 48

34. If 'n' denotes the number of elements in a field, then 'n' must be :

(A) a prime

(B) a product of distinct primes

(C) a prime of the form (4k + 1)

(D) a power of a prime

35. Any finite subgroup of the multiplicative group of a field is :

(A) abelian but not cyclic

(B) cyclic

(C) Nonabelian

(D) is of order 4k + 1; $k \in Z$

36. A bag contains 3 black, 4 white and 2 red balls, all the balls being differnt. The number of selections of at at most 6 balls containing balls of all the colors is :

(A) $4^2 (4!)$

(B) $2^6 \times 4!$

(C) $(2^6 - 1)(4!)$

(D) None of these

37. Which one is not true ?

(A) $\{1, \sqrt{3}\}$ is a basis for $Q(\sqrt{3}, \sqrt{5})$ over $Q(\sqrt{5})$.

(B) $\{1, \sqrt{5}\}$ is a basis for $Q(\sqrt{5})$ over Q.

(C) $\{1, \sqrt{3}, \sqrt{5}, \sqrt{15}\}$ is a basis for $Q(\sqrt{3}, \sqrt{5})$ over Q.

(D) $\{1, -\sqrt{3}, \sqrt{5}, -\sqrt{15}\}$ is a basis of $Q(\sqrt{5}, \sqrt{3})$ over Q.

38. (i) $x^4 + 1$ is irreducible over Z_p for nay prime p

(ii) The ideal $<x^2 + 1>$ is prime in Z[x] but not maximal in Z[x]

(iii) $Z_2[x]/<x^3 + x + 1>$ is a field of order 8

(A) only (i) is correct

(B) only (ii) is correct

(C) only (iii) is correct

(D) (i), (ii) and (iii) are correct

39. Which one of the following statement is not correct :

(A) Z_{31}, the ring of integer modulo 31, is a field.

(B) The set of rationals is a field.

(C) R[x] is not an integral domain.

(D) R[x], the set of polynomials over the set of real numbers is not a field.

40. Let $Y \subset X$; let X and Y are connected topological spaces. It subsets A and B form a separation of $X - Y$ then :

(A) $Y \cup Z$ is connected but $Y \cup B$ is not connected.

(B) $Y \cup B$ is connected but $Y \cup A$ is not connected.

(C) $Y \cup B$ and $Y \cup A$ both are not connected.

(D) $Y \cup A$ as well as $Y \cup B$ are connected.

41. If the differential equation of first order and first degree is of the form $x^{\alpha}y^{\beta}$ (mydx + nxdy)=0 then the integrating factor is :

(A) $x^{km} y^{kn}$, where k can have any value.

(B) $x^{km-\alpha} y^{kn-\beta}$, where k can have any value.

(C) $x^{km+\alpha} y^{kn+\beta}$, where k can have any value.

(D) $x^{km-1-\alpha} y^{kn-1-\beta}$, where k can have any value.

42. Solution of the homogeneous linear equation :

$$x^2\frac{d^2y}{dx^2}-(2m-1)x\frac{dy}{dx}+(m^2+n^2)y=n^2x^m\log x$$

is :

(A) $y=x^m\{C_1\cos(\log x)+C_2\sin(\log x)\}+(\log x)x^m$, where C_1 and C_2 are arbitray constants of integration.

(B) $y=x^m\{C_1\cos(n\log x)+C_2\sin(n\log x)\}+n^2(\log x)x^m$, where C_1 and C_2 are arbitray constants of integration.

(C) $y=n^2x^m\{C_1\cos(n\log x)+C_2\sin(n\log x)\}+(\log x)x^m$, where C_1 and C_2 are arbitray constants of integration.

(D) $y=x^m\{C_1\cos(n\log x)+C_2\sin(n\log x)\}+(\log x)x^m$, where C_1 and C_2 are arbitray constants of integration.

43. If U = constant and V = constant be two independent integrals of equations :

$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$, then

ϕ(U, V) = 0 satisfies the equation

(A) $Pdx+Qdy+Rdz=0$

(B) $Pdx+Qdy=Rdz$

(C) $P\frac{\partial u}{\partial x}+Q\frac{\partial u}{\partial y}+R\frac{\partial u}{\partial z}=0$

(D) $P\frac{\partial u}{\partial x}+Q\frac{\partial u}{\partial y}=R\frac{\partial u}{\partial z}$

44. Solution of the boundary value problem $\frac{\partial u}{\partial x} = 4\left(\frac{\partial u}{\partial y}\right)$ if :

$u(0, y) = 8e^{-3y} + 4e^{-5y}$, is :

(A) $u(x,y) = 8e^{3(4x+y)} + 4e^{5(4x+y)}$

(B) $u(x,y) = 8e^{-3(4x+y)} + 4e^{5(4x+y)}$

(C) $u(x,y) = 8e^{-3(4x+y)} + 4e^{-5(4x+y)}$

(D) $u(x,y) = 8e^{3(4x+y)} + 4e^{-5(4x+y)}$

45. The solution of :

$\frac{\partial u}{\partial x} = 2\left(\frac{\partial u}{\partial t}\right) + u$, where $u(x, 0) = 6e^{-3x}$

by the method of separation of variables is :

(A) $u(x, t) = 6e^{-(3x+t)}$

(B) $u(x, t) = 6e^{-(3x+2t)}$

(C) $u(x, t) = 6e^{-(3x+3t)}$

(D) $u(x, t) = 6e^{-(3x+4t)}$

46. The complete integral of

$yz\,p^2 - q = 0$

by Charpit's method is :

(A) $z^2(a+y^2) = (x + b)$; a, b being arbitray constants

(B) $z^2(a^2 + y^2) = (x + b)$; a, b being arbitrary constants

(C) $z^2(a^2 - y^2) = (x + b)$; a, b being arbitrary constants

(D) $z^2(a^2 - y^2) = (x + b)^2$; a, b being arbitrary constants

47. If f(x) is polynomial of degree n in x then n^{th} difference of this polynomial is :

(A) Constant

(B) Variable

(C) Zero

(D) None of these

48. The value of $\sqrt{8}$ by Newton-Rephson method is approximately :

(A) 1.8285

(B) 2.8285

(C) 0.8285

(D) 3.8285

49. The approximate value of :

$$y = \int_0^{\pi} \sin x \, dx$$

by Simpson's $\frac{1}{3}$ rule is :

(A) 2

(B) 1.9540

(C) 2.0008

(D) 2.3

50. Match the sufficient conditions in the following table for the nature of the extremal of the given functional :

	Nature of the extremal of the given functional		Sufficient Condition
1.	For a weak minimum to be obtained on the extremal on the curve C	a.	$Fy'y' \geq 0$ at points close to C and also for arbitrary values of y'
2.	For a weak maximum to be attained on the extremal on the curve C	b.	$Fy'y' \leq 0$ at points close to C and also for arbitrary values of y'
3.	For a strong minimum to be attained on the extremal on the curve C	c.	$Fy'y' > 0$ on C
4.	For a strong maximum to be attained on the the curve C	d.	$Fy'y' < 0$ on C

where the symbols have their usual meanings :

Codes the answer from the codes given below :

	1	2	3	4
(A)	a	b	c	d
(B)	b	a	d	c
(C)	c	d	a	b
(D)	d	c	b	a

51. Solution of the integral equation $y(x) = f(x) + \lambda \int_0^1 e^{a(x^2-t^2)} y(t)\, dt$, by computing the resolvent Kernel, is :

(A) $y(x) = f(x) + \lambda \int_0^1 e^{a(x^2-t^2)} f(t)\, dt$

(B) $y(x) = f(x) + \frac{\lambda}{1+\lambda} \int_0^1 e^{a(x^2-t^2)} f(t)\, dt$

(C) $y(x) = f(x) + \frac{\lambda}{1-\lambda} \int_0^1 e^{a(x^2-t^2)} f(t)\, dt$

(D) $y(x) = f(x) + \frac{1-\lambda}{\lambda} \int_0^1 e^{a(x^2-t^2)} f(t)\, dt$

52. Lagrange's equations for a holonomic dynamic system specified by n generalized co-ordinates q_j (j = 1, ... n) are :

(A) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j,\ (j = 1, 2, ... n)$

(B) $\frac{d}{dt}\left(\frac{\partial T}{\partial q_j}\right) - \frac{\partial T}{\partial q_j} = Q_j,\ (j = 1, 2, .n)$

(C) $\frac{d}{dt}\left(\frac{\partial T}{\partial q_j}\right) - \frac{\partial T}{\partial \dot{q}_j} = Q_j,\ (j = 1, 2, ... n)$

(D) $\frac{d}{dt}\left(\frac{\partial T}{\partial q_j}\right) + \frac{\partial T}{\partial \dot{q}_j} = Q_j,\ (j = 1, 2, .. n)$

53. Which one of the following is represented by Hamiltonian function H for conservative system, in case the co-ordinate transformation is independent of time ?

(A) Kinetic Energy
(B) Potential Energy
(C) Total Energy
(D) None of these

54. The Lagrangian L for a particle moving under the influence of a central force $\vec{F} = -\frac{k\vec{r}}{r^3}$ is :

(A) $\frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{k}{r}$
(B) $\frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{k}{r}$
(C) $\frac{m}{2}(\dot{r}^2 - r^2\dot{\theta}^2) + \frac{k}{r}$
(D) $\frac{m}{2}(\dot{r}^2 - r^2\dot{\theta}^2) - \frac{k}{r}$

55. When an impulse is aplied to a rigid body, changes are produced in :

(A) Both linear momentum and moment of momentum
(B) Linear momentum but not in moment of momentum
(C) Moment of momentum but not is linear momentum
(D) Neither linear momentum nor moment of momentum

56. A random variable assumes values –1, 0, 1 and 2 with probabilities

$\frac{1+3\alpha}{4}, \frac{1-\alpha}{4}, \frac{1+2\alpha}{4}$ and $\frac{1-4\alpha}{4}$

respectively. What among the following is the most appropriate value of α.

(A) $\alpha = 1$
(B) $-\frac{1}{2} \le \alpha \le \frac{1}{4}$
(C) $-\frac{1}{3} \le \alpha \le \frac{1}{4}$
(D) None of these

57. A stratified random sample of size 120 to be selected from a population of size 800. The sizes of strata and their standard deviations are given below :

Strata	I	II	III
Sizes	200	300	300
Standard Deviations	6	8	12

The sizes of the sample from second and third stratum are :

(A) 40, 40 (B) 45, 45

(C) 40, 60 (D) 60, 40

58. If all the correlation coefficients of zero order in a st of p-variates are equal to r then every partial correlation of s^{th} order is equal to :

(A) sr (B) $\frac{sr}{1+r}$

(C) $\frac{r}{1+sr}$ (D) $\frac{1+sr}{r}$

59. Which of the following statements for randomized block design (RBD) is **not** correct ?

(A) RBD is the most suitable design when experimental material is heterogeneous and homogeneous blocks can be formed.

(B) In RBD, equal number of replications are alloted to each treatment.

(C) The analysis in RBD can be carried out even if observations on entire block or treatment are missing.

(D) The RBD is the most suitable design fro testing a large number of treatments.

60. The time to failure of a component of an electronic device has an exponential distribution with median of 20 hours. What is the probability that the component will work without failing for at least 40 hours ?

(A) $\frac{1}{2}$

(B) e^{-800}

(C) $1 - e^{-800}$

(D) $\frac{1}{4}$

61. Let $X_1, X_2 \ldots\ldots X_n$ be a random sample from $N(\mu, \sigma^2)$ and if

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad s^2 = \frac{1}{n-1}\Sigma\left(X_i - \bar{X}\right)^2$$

then which of the following statements is incorrect ?

(A) $\bar{X}$ and s^2 are independently distributed.

(B) $\bar{X}$ is an unbiased estimator of μ.

(C) s^2 is an unbiased estimator of σ^2.

(D) s^2 is maximum likelihood estimator of σ^2.

62. The maximum likelihood estimate of θ from a distribution with probability density function

$$f(x,\theta) = \begin{cases} e^{-(x-\theta)} & ; x \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

based on the random sample $X_1, \ldots.. X_n$ is

(A) Sample mean $\bar{X}$

(B) Max $(X_1, \ldots\ldots X_n)$

(C) Min $(X_1, \ldots\ldots X_n)$

(D) Does not exist.

63. Let $X_1, X_2, \ldots\ldots X_n$ be a random sample from $N(\mu, 1)$ where μ is inknworn. The sampling distribution of $\sum_{i=1}^{n}(x_i - \overline{x})^2$ is :

(A) Chi square with n degrees of freedom

(B) Chi square with (n – 1) degrees of freedom

(C) Students t with n degrees of freedom

(D) Students t with (n – 1) degrees of freedom

64. A test paper has 100 questions, all of multiple choice type. For each question, the student has a choice of five answers out of which he has to choose one correct answer. Each correct answer carries 1 mark and each incorrect answer carries a penalty of $\frac{1}{4}$ marks. What is the expected total marks of the candidate if he chooses the answer to each question at random ?

(A) 0 (B) –25

(C) 25 (D) 100

65. The mode and harmonic mean of the random variable X with probability density function

$$f(x) = \begin{cases} 6x(1-x) & ; 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

are respectively :

(A) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (B) $\left(\frac{1}{3}, \frac{1}{2}\right)$

(C) $\left(\frac{1}{6}, \frac{1}{2}\right)$ (D) $\left(\frac{1}{2}, 3\right)$

66. In order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = 3/4$, a coin is tossed five times and H_0 is rejected if more than 3 heads are obtained. The size of critical region for this test is :

(A) $\frac{3}{16}$

(B) $\frac{3}{32}$

(C) $\frac{1}{4}$

(D) $\frac{1}{16}$

67. In stratified random sampling, for a simple cost function, $V(\bar{y}_{st})$ is minimized according to Neymann allocation if the size of h^{th} stratum is proportional to :

(A) N_h

(B) $N_h S_h$

(C) $\frac{N_h}{S_h}$

(D) $N_h S_h^2$

68. In the p variate linear regression model

$$y_{n\times 1} = X_{n\times p}\beta_{p\times 1} + u_{n\times 1}; u \sim N(0, \sigma^2 I)$$

which of the following statements is incorrect ?

(A) $b = (X'X)^{-1} X'y$ is the ordinary least squares estimator of β

(B) b is the maximum likelihood estimator of β

(C) The errors are nonspherical

(D) b is the consistent estimator of β iff plim $\left(\frac{1}{n}X'u\right) = 0$

69. Multiple comparison Tests are carried out once the null hypothesis in analysis of variance is rejected. Some tests are listed below :

(1) Tukey's Test

(2) Bartlett's Test

(3) Duncan's multiple range Test

(4) Neumann Keul's Test

(A) (1), (2) and (3) only are multiple comparison Tests

(B) (1), (3) and (4) only are multiple comparison Tests

(C) (1), (2) and (4) only are multiple comparison Tests

(D) All the above tests are multiple comparison Tests

70. Let $X_1, X_2 \ldots\ldots X_n$ be a sequence of independently and identically distributed random variables

with $\rho(X_i = -1) = \rho(X_i = 1) = \frac{1}{2}$

Let Z be a standard normal variate with $\rho(-0.1 \le z \le 0.1) = 0.08$

Define $S_n = \sum_{i=1}^{n} X_i$ then

$\lim_{n \to \infty} \rho\left(S_n > \frac{n}{10}\right)$ is :

(A) 0.92
(B) 0.46
(C) 0.04
(D) 0.08

71. The rainfall in a certain region is a normally distributed random variable with mean 40 cm and variance 4 cm^2. The probability that the rainfall in a particular year lies between 15 cm to 25 cm, will be at least :

(A) 0.16
(B) 0.04
(C) 0.84
(D) 0.96

72. In large samples with simple random sampling, the ratio estimate $\hat{Y}_R$ has smaller variance than $\hat{Y} = N\bar{y}$ if the correlation ρ between Y and X is :

(A) $> \frac{1}{2} \frac{\text{coeff of var (y)}}{\text{coeff of var (x)}}$
(B) $= \frac{1}{2} \frac{\text{coeff of var (y)}}{\text{coeff of var (x)}}$
(C) $< \frac{1}{2} \frac{\text{coeff of var (y)}}{\text{coeff of var (x)}}$
(D) $= \frac{1}{2} \frac{\text{coeff of var (y)}}{\text{coeff of var (x)}}$

73. The characteristic function of a random variable X is given by

$$\varnothing_x(t) = \left(1 - i\frac{t}{\lambda}\right)^{-p}$$

The mean and variance of the variable X are :

(A) $\frac{\lambda}{p}, \frac{\lambda}{p^2}$ (B) $\frac{p}{\lambda}, \frac{p}{\lambda^2}$

(C) $\frac{p}{\lambda}, \frac{p^2}{\lambda^2}$ (D) $\frac{\lambda}{p}, \frac{\lambda^2}{p^2}$

74. Two systems S_1 and S_2 are connected in parallel, each having exponential failure rate with parameter λ. The failure distribution of the overall system is given by :

(A) $f(t) = 1 - e^{-\lambda t}$ (B) $f(t) = e^{-\lambda t}\left(1 - e^{-\lambda t}\right)$

(C) $f(t) = 2\lambda e^{-\lambda t}\left(1 - e^{-\lambda t}\right)$ (D) $f(t) = \frac{1}{2}\lambda e^{-\lambda t}\left(1 - e^{-\lambda t}\right)$

75. The following data pertains to liking of tea and gender of person

	Like Tea	Dislike Tea
Male	12	13
Female	8	7

The null hypothesis that the taste for tea is independent of the gender of the person is to be tested. If tabulated $x^2_{(.05, 1)} = 3.84$,

Read the following satatements and tick the correct option :

(1) The null hypothesis is accepted.

(2) The test is carried out using 1 degrees of freedom.

(A) Only statement (1) is correct. (B) Only statement (2) is correct.

(C) Both the statements (1) and (2) are correct. (D) Both the statements (1) and (2) are incorrect.